046 **HIGH SPEED BEAMFORMING ON ARBITRARY ORTHOGONAL COORDINATE SYSTEM THROUGH FOURIER'S TRANSFORM WITH NO APPROXIMATE CALCULATIONS.**

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Summary: A new wavenumber matching method to be used in Fourier's transform-based beamforming methods is reported. Usually, the wavenumber matching is performed approximately by interpolating spectra in a frequency domain or using a special Fourier's transform with nonuniform sampling intervals. In contrast, the new wavenumber matching method permits to carry out beam steering on an arbitrary coordinate system without any approximate calculations. For echo data generation, a high accuracy and a high speed are achieved. The feasibility of the new wavenumber matching method is confirmed for J.-y. Lu's method through simulations and agar phantom experiments.

1. Introduction

Recently, raid tissue motions are measured ultrasonically with high frame rates in various manners (e.g., parallel processing etc.), for instance, shear wave propagation, carotid blood flow, heart motion, use of a 2-dimensional (2D) arrayed-type transducer etc. For such measurements, we previously proposed to use steered plane wave transmissions (e.g., [1]) in conjunction with the use of the lateral modulation (LM) method (e.g., [2]) and a steering angle (ASTA) method (e.g., [3]). The transmissions are performed respectively or simultaneously. The respectively received echo data with different steering angles can also be coherently superposed to yield high lateral resolution echo data (e.g., [1]). For such plane wave generations, as far, other three groups from us have reported fast beamforming methods using Fourier's transforms ([4-6]).

In this study, a new wavenumber matching method to be used in Fourier's transform-based beamforming methods is reported [7,8]. In such Fourier's transform-based beamfroming methods, the wavenumber matching is usually performed approximately by interpolating spectra in a frequency domain [4,6] or using a special Fourier's transform with nonuniform sampling intervals [5]. In contrast, the new wavenumber matching method permits to carry out beam steering on an arbitrary coordinate system without any approximate calculations. For echo data generation, a high accuracy and a high speed are achieved. The feasibility of the new wavenumber matching method was confirmed for J.-y. Lu's method [4] through simulations and agar phantom experiments.

2. Methods

To achieve the steering shown in Fig. 1, the new wavenumber matching is performed with respect to J.-y. Lu's method [4] through the two complex exponential multiplication steps (see steps 2 and 4 shown in Fig. 2) for the respective lateral (x) and depth (y) directions [7,8]. The step 4 also performs the calculation of angular spectra at respective depths simultaneously (i.e., obtaining a resolution) [7]. That is, before performing a lateral (x) Fast Fourier's transform (FFT), signals are multiplied by $\exp(ik_x^t x) = \exp(ik \sin\theta x)$ where k is the wavenumber of ultrasound and when non-steering is performed, θ is 0° (actually, FFT including the multiplication is effective); and when calculating the angular spectra of respective depths of interest by multiplying $\exp\left(i\sqrt{k^2 - k_x^2}\, y\right)$ [7], signals are multiplied by $\exp(ik_y^t y) = \exp(ik \cos\theta y)$ simultaneously.

The calculation is similar to the classical synthetic aperture (SA) method reported by Busse [9]. Although these calculations are performed through the conventional one-dimensional Fast Fourier's transform (1D FFT) in steps 1 and 3, the last two-dimensional (2D) inverse Fourier's transform to be performed is carried out by the faster processing reported by Busse [9] (steps 5 and 6), i.e., the summation of spectra with respect to a wavenumber for the respective lateral wave numbers and only 1D IFFT at the respective depths.

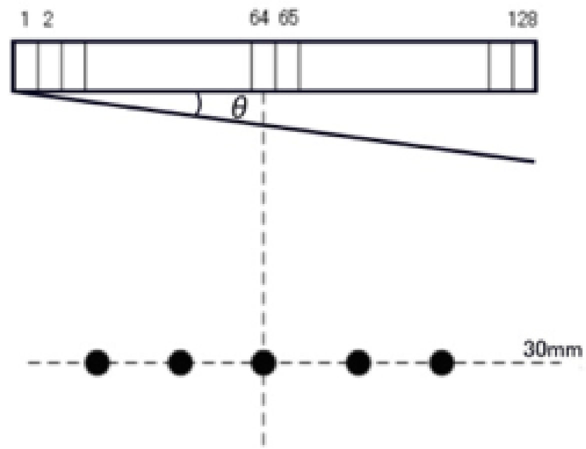


Fig. 1. Beam steering with steering angle θ. The 128 linear array elements and 5 scatters (depth 30 mm) are also depicted to be used in simulations.

1. FFT with respect to time t; and performing down-sampling if possible.
2. Matching on lateral kx.
3. FFT with respect to lateral x.
4. Matching on depth ky and calculation of angular spectra with respective depths of interest.
5. Summation with respect to k.

Fig. 2. Steps of beamforming.

Also for a dynamic focusing, such special FFTs will be effective to complete calculations faster.

When plural steered plane waves are transmitted simultaneously, respective echo waves separated in a frequency domain in advance are similarly processed, and finally the steps 5 and 6 are performed on superposed data.

3. Simulations

Simulations are performed to confirm the feasibility of the new wavenumber matching method. An arrayed-type transducer is simulated using Field II [10]. For instance, with respect to the simulation phantom (5 scatters at a depth 30 mm) shown in Fig. 2, a linear arrayed-type transducer (3 MHz; pitch, 0.1 mm; kerf, 0.025 mm; 128 elements) yields echo data with no steering (Fig. 3a) and with steering angles, 5°, 10° and 20°(Fig. 3b-3d). Generated steering angles were statistically evaluated using spectral moments [11] (Table I). Regardless the steering angles 0° to 20°, errors in a generated steering angle are less than 1°.

Although the superposition of a few echo data frames with a coarse steering angle interval permits both echo imaging with a high lateral resolution and accurate displacement measurement (for instance, lateral wavelengths, 2λ, 3λ, $\infty\lambda$ etc. [1]), Fig. 4 shows echo data obtained with a fine steering angle interval $\Delta\theta = 1^\circ$: (a) $\theta = 0^\circ \pm 5^\circ$ (i.e., 11 waves), (b) $0^\circ \pm 10^\circ$ (21 waves) and (c) $0^\circ \pm 20^\circ$ (41 waves). Increasing the number of waves increased a lateral resolution. Practically, the range of a steering angle and an echo SNR are critical together with the number of waves. Fig. 4c shows echo data obtained using Busse's SA method (θ =0°). Note that the Busse's SA method yields a high lateral resolution.

4. Agar phantom experiments

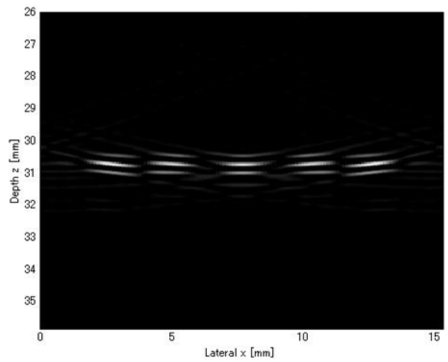
Agar phantom SA data were also used for the new method. Fig. 5 shows envelope-detected echo images obtained for steering angles, (a) 0°, (b) 5°, (c) 10° and (d) 20°.

5. Conclusions

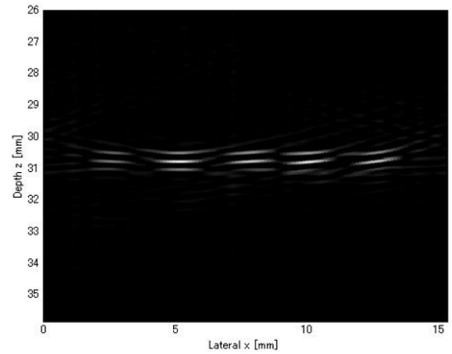
We developed a new wavenumber matching method to be performed in high speed Fourier's transform-based beamforming for plane wave transmissions. The feasibility and accuracy were confirmed

Table I. Generated steering angles for simulated phantom.

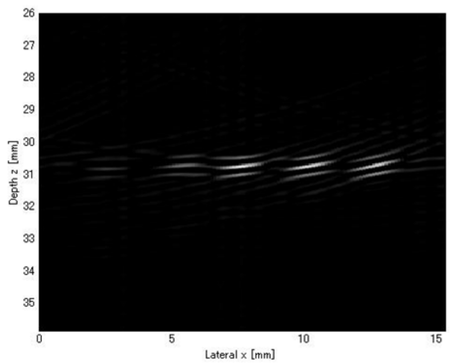
Set steering angle [deg]	0	5	10	15	20
Estimated angle [deg]	0.1	4.6	9.2	14.7	19



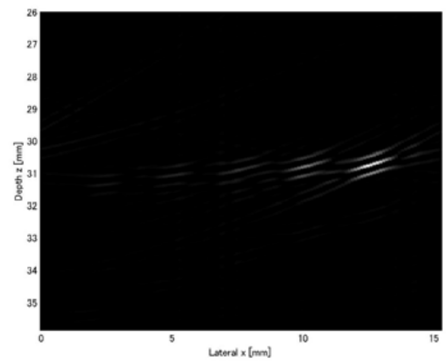
(a)



(b)

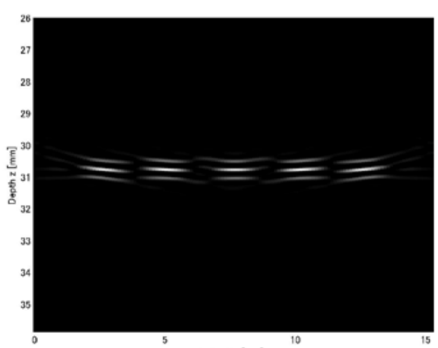


(c)

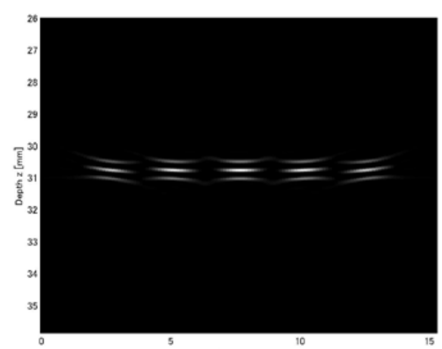


(d)

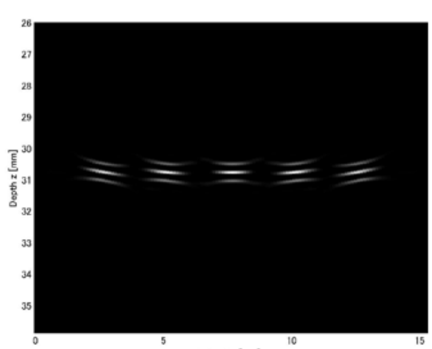
Fig. 3. Echo (rf) images obtained (a) with non-steering (steering angle, θ =0°), and θ = (b) 5°, (c) 10° and (d) 20°.



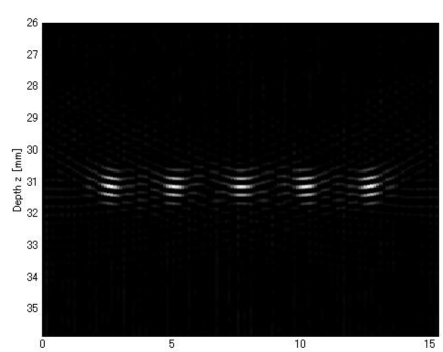
(a)



(b)



(c)



(d)

(continue.)

Fig. 4. Echo (rf) images obtained via coherent superposition of waves (a) with θ =0° $\pm 5^{\circ}$ (11 waves with interval, $\Delta\theta = 1^{\circ}$), (b) 0° $\pm 10^{\circ}$ (21 waves) and (c) 0° $\pm 20^{\circ}$ (41 waves). (d) Image obtained using Busse's SA method (θ =0°).

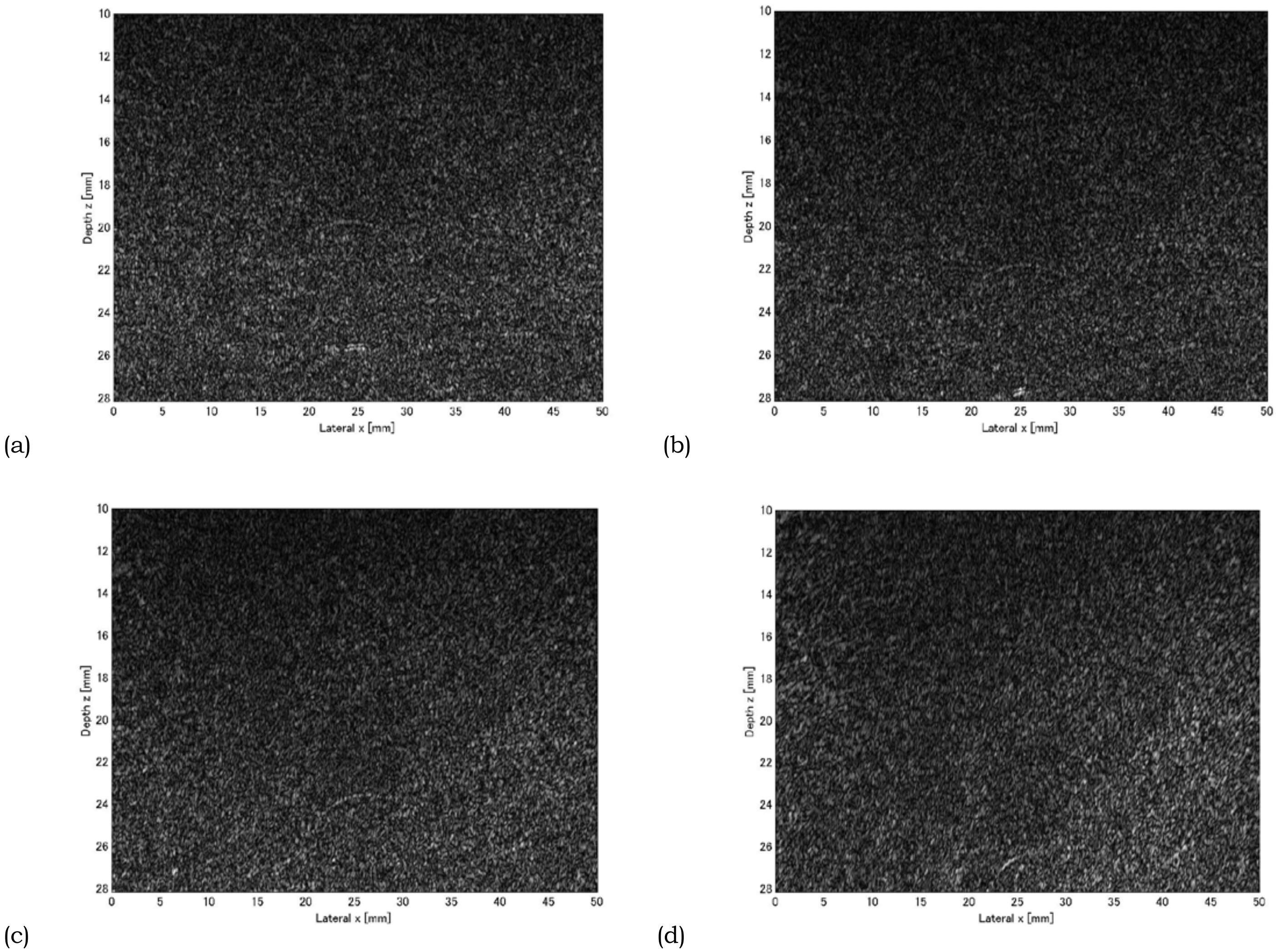


Fig. 5. Envelope-detected images obtained for agar phantoms for steering angles, (a) 0°, (b) 5°, (c) 10° and (d) 20°.

through simulations and agar phantom experiments. Particularly in this report, the feasibility of the new wavenumber matching method was confirmed for J.-y. Lu's method. Different from other Fourier's transformed-based methods, the method permitted the echo generation without performing any approximations on an arbitrary orthogonal coordinate system. The new method also achieved both non-steering and steering without any approximate calculations. The PSFs obtained were compared with that obtained using Busse's SA method. Coherent superposition of steered waves was effective to increase a lateral resolution. The method can also be applied to other Fourier's methods including migration methods (e.g., [6]). Using the new method, various scanning can be performed with a high accuracy (e.g., sector, radial, convex, fan etc.). The accuracy of echo imaging and displacement measurements will also be improved.

Appendix. Rotation of coordinate system

A coordinate system can be rotated by delaying SA data at respective lateral position x via multiplication of $\exp[-j\omega\tau(x)]$ in a frequency domain (Fig. A1) [12].

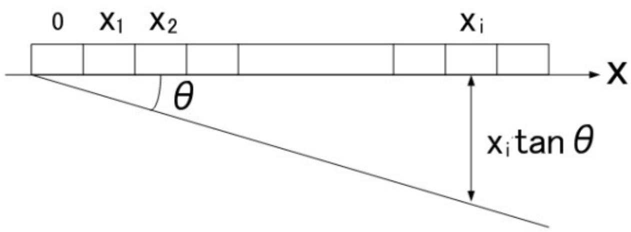


Fig. A1. Rotation of coordinate system.

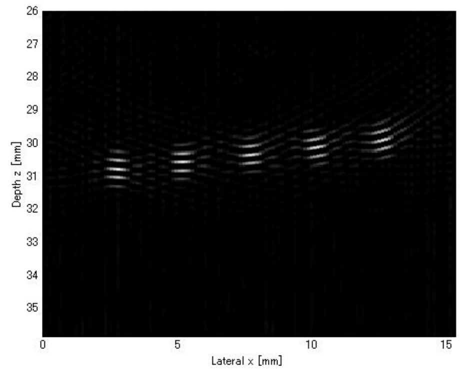


Fig. A2. Rotation of coordinate system by 5°.

$$\tau(x) = 2 \times \frac{x \tan \theta}{c}.$$

Using the same transducer parameters, the same simulated phantom was dealt with. Fig. A2 shows the 5° rotation achieved using Busse's method. A rotation angle obtained was 4.6°.

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